Computing Nash Equilibria in Timed Games

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under supervision of Patricia Bouyer and Nicolas Markey

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Motivations

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- Instead of the worst case, we consider a rational environment
- Equilibria notions capture rational behaviors of players

Definition

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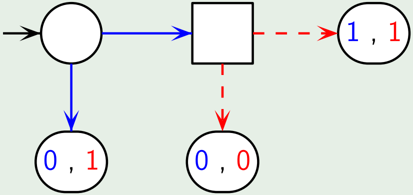
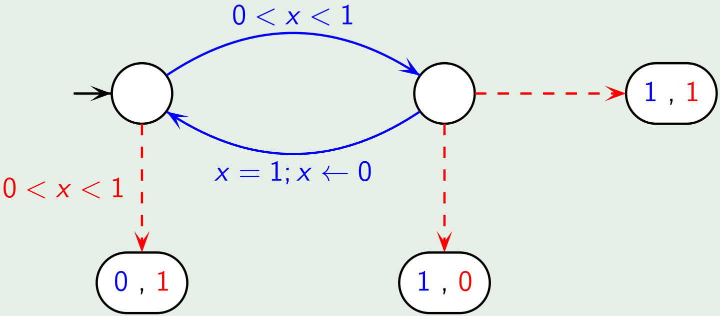
Game on graphs



Payoff functions we consider :

- ω-regular
- based on an observation variable

Timed Game Example



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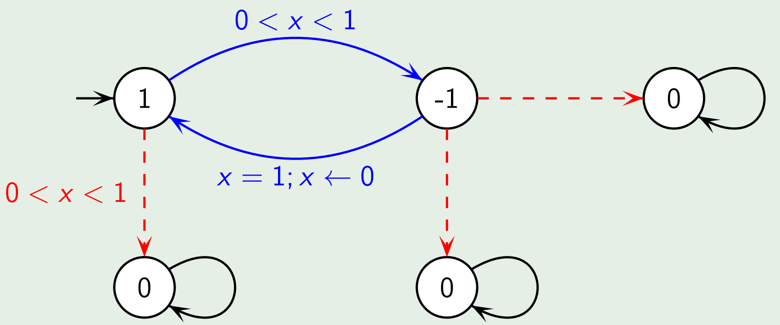
Weighted Timed Game Example



Our Results

- The existence of a Nash equilibrium in weighted timed games is undecidable
- For qualitative objectives we obtain decidability results using algorithms based on the region graph

Game with no equilibria

$$payoff = (cost, -cost)$$

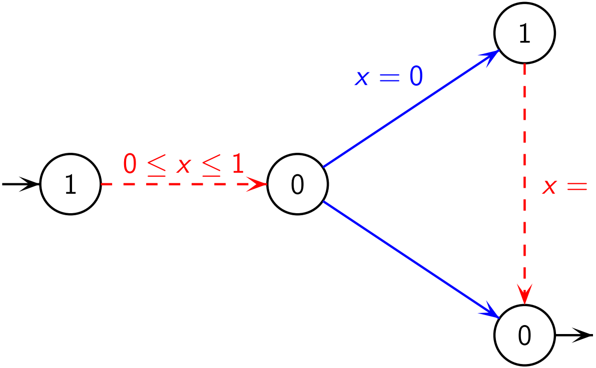


Figure: Mod_ϵ

Encoding a counter machine

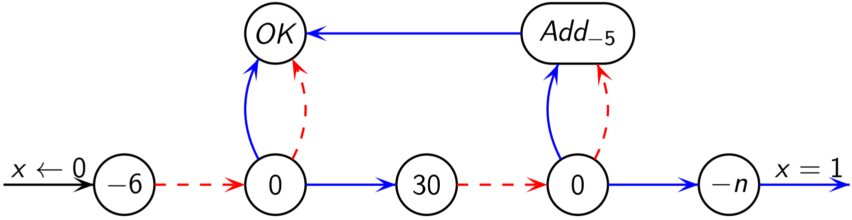


Figure: Mod_n

- $n = 3$: increment c1
- $n = 2$: increment c2
- $n = 12$: decrement c1
- $n = 18$: decrement c2

$$E(c_1, c_2) = 5 - \frac{1}{2^{c_1} \cdot 3^{c_2}}$$

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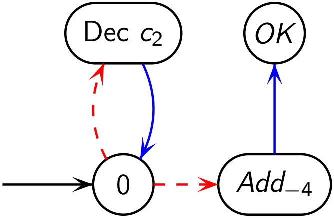


Figure: $\text{Test}(c_1 = 0)$

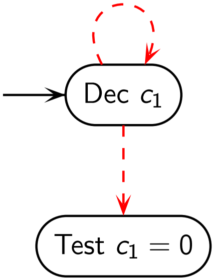


Figure: $\text{Test}(c_1 > 0)$

Encoding a counter machine

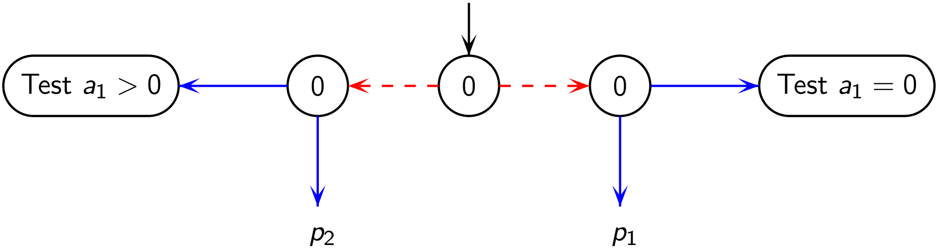


Figure: Module Cond ($c_1 = 0$)

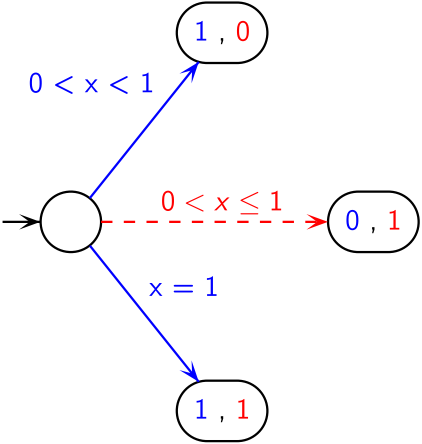
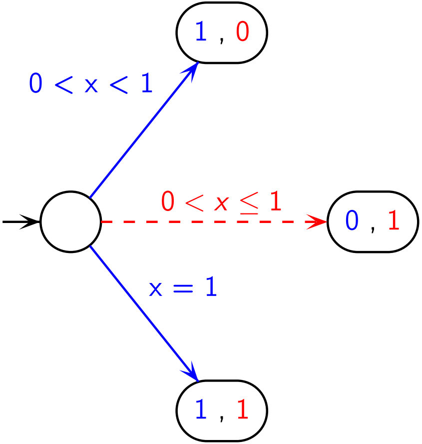


Figure: Timed Game G



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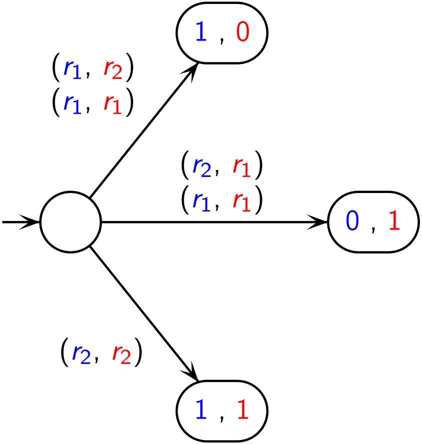
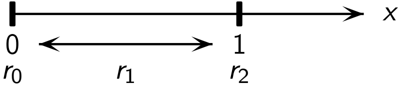
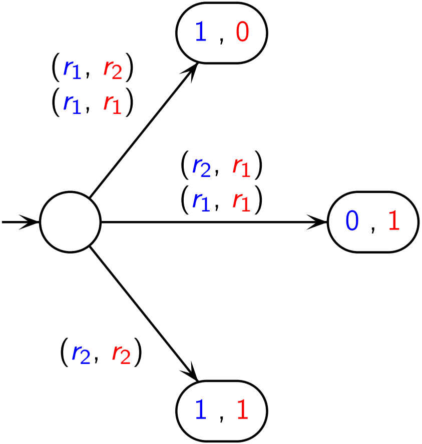


Figure: Region Game $R(G)$





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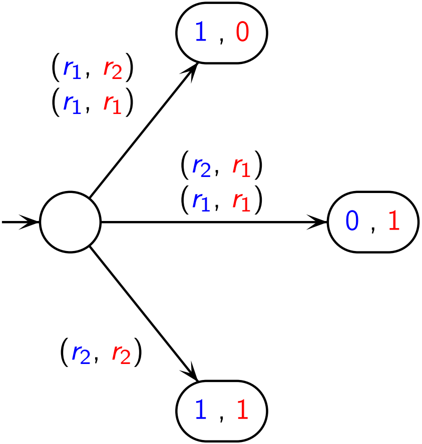

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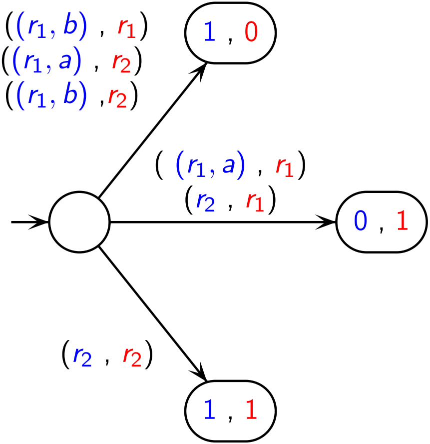
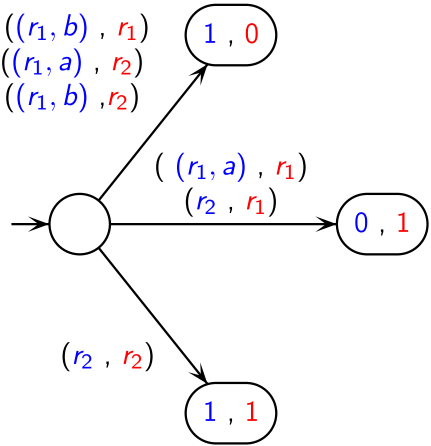


Figure: Concurrent Game $C_1(G)$

b : *before*

a : *after*



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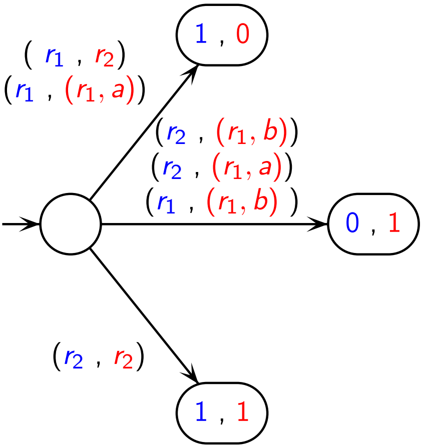
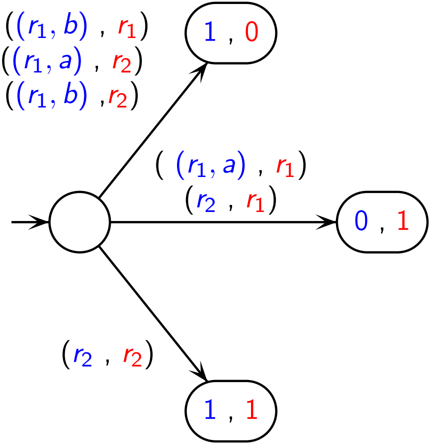


Figure: Concurrent Game $C_2(G)$



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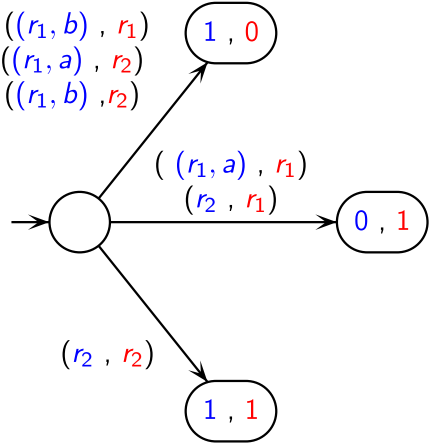

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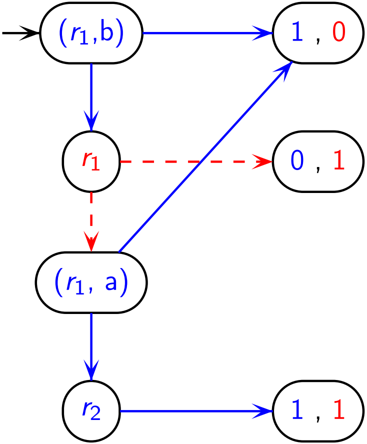
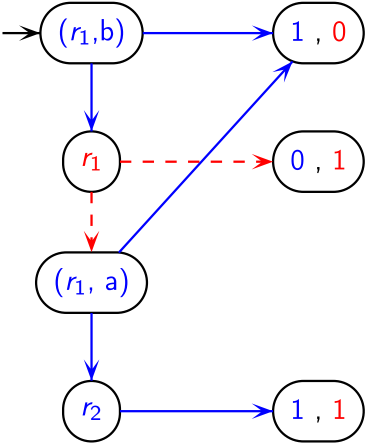


Figure: Turn Based Game $T_1(G)$



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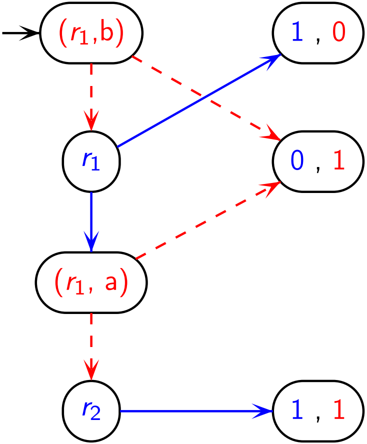


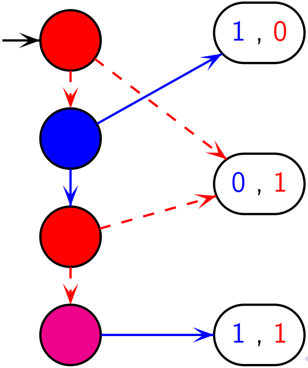
Figure: Turn Based Game $T_2(G)$

Theorem

There exists a Nash equilibrium in the timed game if and only if there exists a "twin" Nash equilibrium in the two turn based games

Purely qualitative case

- If there is a path with payoff $(1, 1)$ then it is a Nash equilibrium



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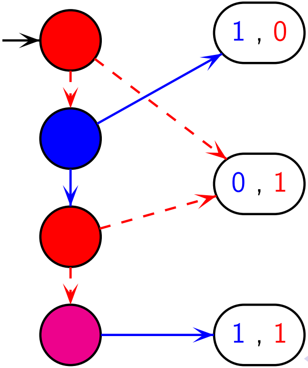
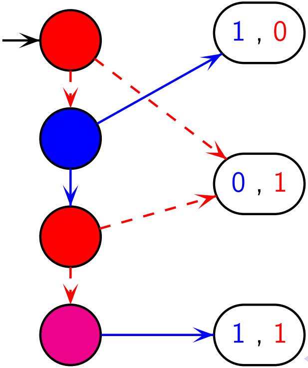
- There is a Nash equilibrium with payoff $(0, 0)$ if no player has a winning strategy in the game where he has the advantage



Purely qualitative case

- If there is a path with payoff (1, 1) then it is a Nash equilibrium
- There is a Nash equilibrium with payoff (0, 0) if no player has a winning strategy in the game where he has the advantage
- If there is a Nash equilibrium with payoff (1, 0) in the game T_2, there is a twin Nash equilibrium in the two turn based games.



Perspectives

- Computing twin equilibria using tree automata
- N-players games
- Quantitatives objectives
- Other kinds of equilibria (subgame perfect equilibria, . . .)