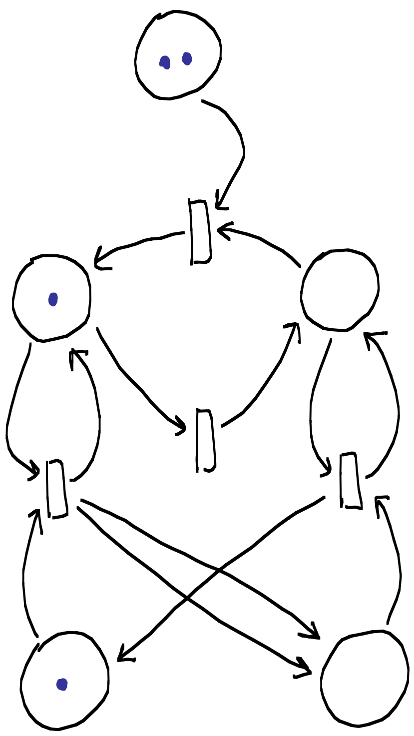
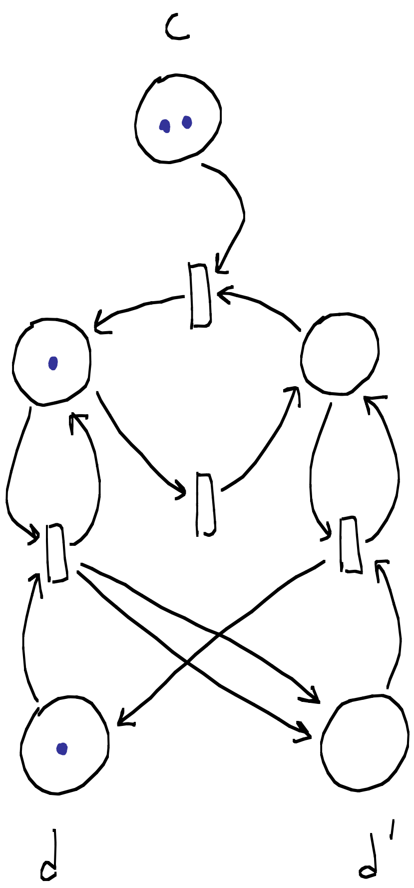
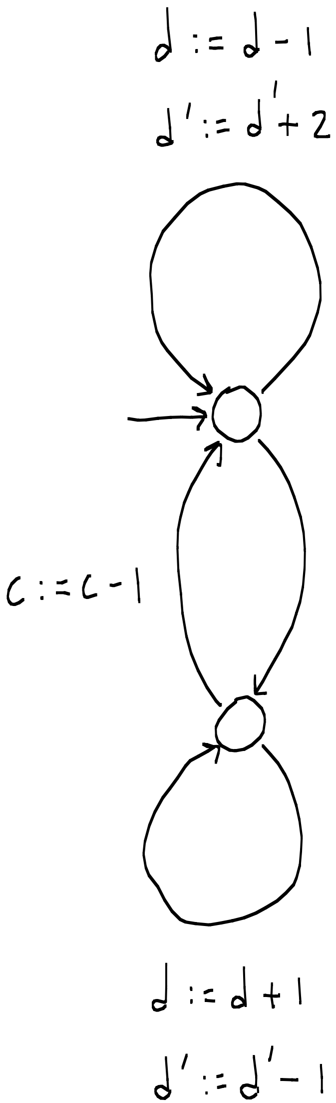
Petri net



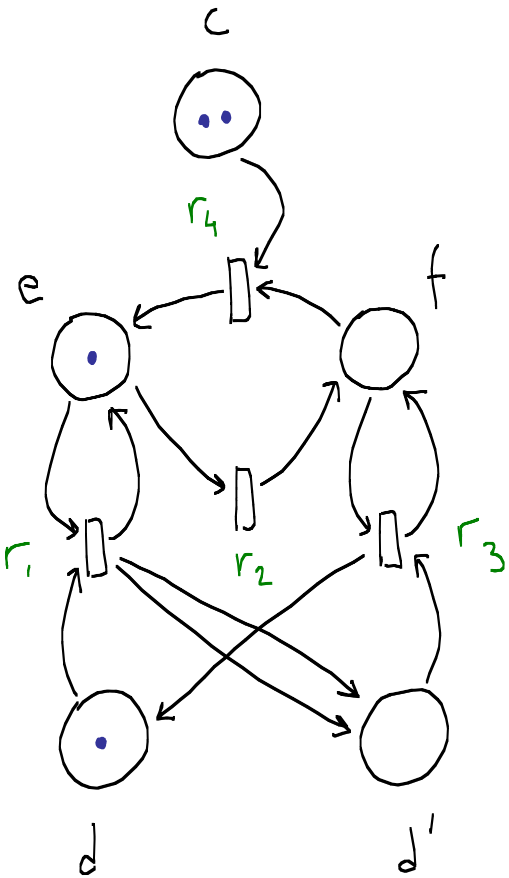
Petri net



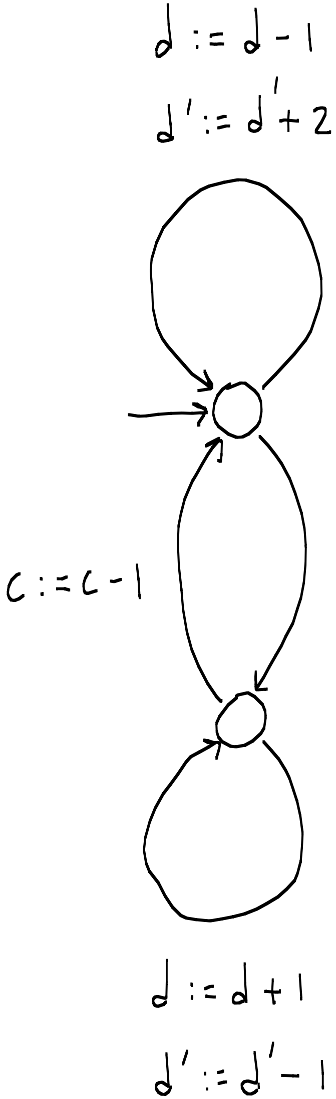
Counter machine



Petri net



Counter machine



Vector addition system (VAS)

	c	d	d'	e	e'	f	f'
r_1	0	−1	2	−1	1	0	0
r_1'	0	0	0	1	−1	0	0
r_2	0	0	0	−1	0	1	0
r_3	0	1	−1	0	0	−1	1
r_3'	0	0	0	0	0	1	−1
r_4	−1	0	0	1	0	−1	0

Reachability

Given a VAS $R \subseteq^{fin} \mathbb{Z}^k$ and $v, w \in \mathbb{N}^k$,

is there an <u>admissible</u> computation from v to w?

Decidable ...

[Sacerdote & Tenney STOC '77]

[Mayr STOC '81, SIAM JOC '84]

[Kosaraju STOC '82]

[Reutenauer '89]

[Lambert TCS '92]

[Leroux LICS '09]

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... but no primitive recursive algorithm is known!

ExpSpace hardness [Lipton '76]:

Reduction from the halting problem for Minsky machines of size n with counters bounded by 2^{2^n}.

$$c_1 + \overline{c_1} = 2^{2^1} \qquad c_2 + \overline{c_2} = 2^{2^2} \qquad \cdots \qquad c_n + \overline{c_n} = 2^{2^n}$$

$$d_1 + \overline{d_1} = 2^{2^1} \qquad d_2 + \overline{d_2} = 2^{2^2} \qquad \qquad d_n + \overline{d_n} = 2^{2^n}$$

$$\text{inc}(c_{i+1}) \mathrel{\hat{=}} c_{i+1} := c_{i+1} + 1 \; ; \; \overline{c_{i+1}} := \overline{c_{i+1}} - 1$$

$\text{if\_zero}(c_{i+1}) \mathrel{\hat{=}}$ for $c_i := 1$ to 2^{2^i}

for $d_i := 1$ to 2^{2^i}

$\text{inc}(c_{i+1})$

Covering

Given a VAS $R \subseteq^{fin} \mathbb{Z}^k$ and $v, w \in \mathbb{N}^k$,

is there an <u>admissible</u> computation from v to $\geqslant w$?

Boundedness

Given a VAS $R \subseteq^{fin} \mathbb{Z}^k$ and $v \in \mathbb{N}^k$

is the set of all w reachable from v finite ?

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Boundedness

Given a VAS $R \subseteq^{fin} \mathbb{Z}^k$ and $v \in \mathbb{N}^k$

is the set of all w reachable from v finite ?

For Petri nets with resets
and lossy Minsky machines,
covering is not prim. rec.
[Schnoebelen IPL '02]

ExpSpace memberships:

The covering and boundedness problems for vector addition systems

Charles Rackoff

Toronto

TCS '78

The covering and boundedness problems for branching vector addition systems

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TCS '78

The covering and boundedness problems for branching vector addition systems

Stéphane Demri
LSV, ENS Cachan,
CNRS, INRIA Saclay

Marcin Jurdziński
Oded Lachish
Ranko Lazić
} Warwick

FSTTCS '09

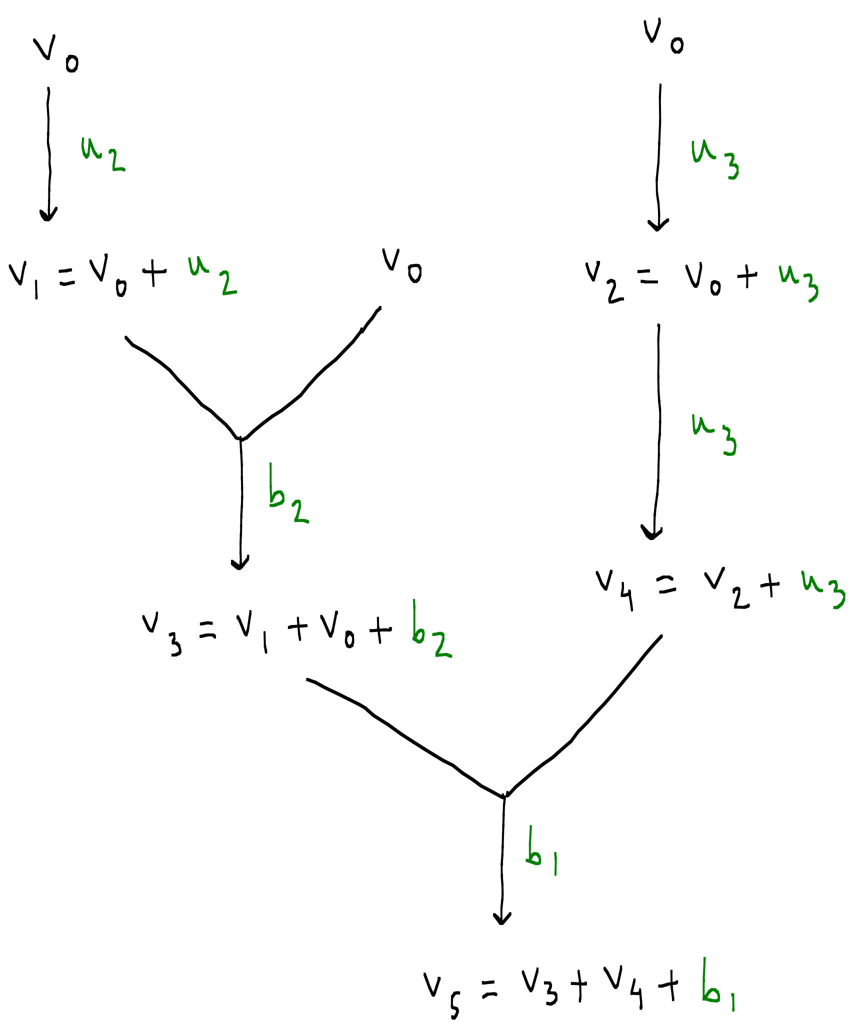
VAS $\quad r_1, r_2, r_3 \in \mathbb{Z}^k$

$$
\begin{aligned}
& v_0 \\
& \downarrow r_2 \\
& v_1 = v_0 + r_2 \\
& \downarrow r_1 \\
& v_2 = v_1 + r_1 \\
& \downarrow r_1 \\
& v_3 = v_2 + r_1
\end{aligned}
$$

BVAS

$u_1, u_2, u_3 \in \mathbb{Z}^k$

$b_1, b_2 \in \mathbb{Z}^k$



Some motivations:

- linear logic

  [de Groote et al. LICS '04]

- tree automata for security

  [Verma & Goubault-Larrecq DMTCS '05]

- FO on data trees

  [Bojańczyk et al. J. ACM '09]

- computational linguistics

  [Rambow ACL '94]

The covering and boundedness problems for branching vector addition systems

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2 ExpTime complete !

The covering and boundedness problems
for vector addition systems
branching

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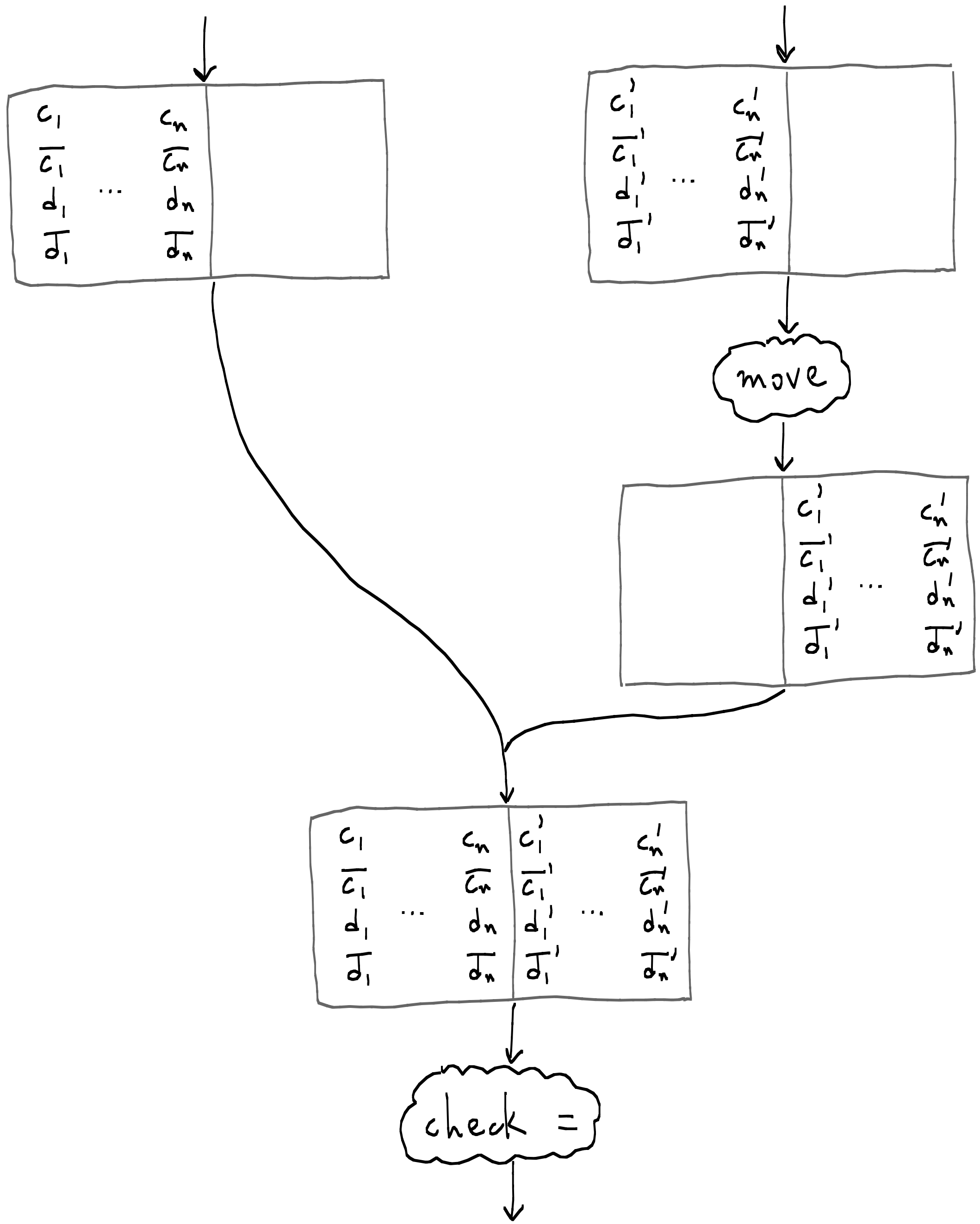
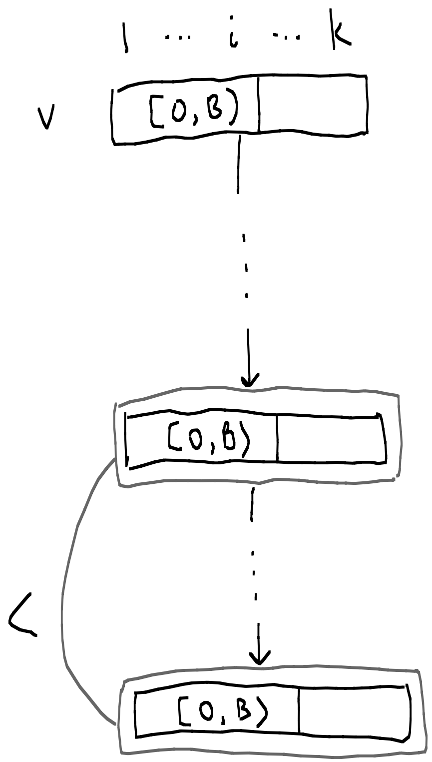
Adding alternation to Lipton's construction :



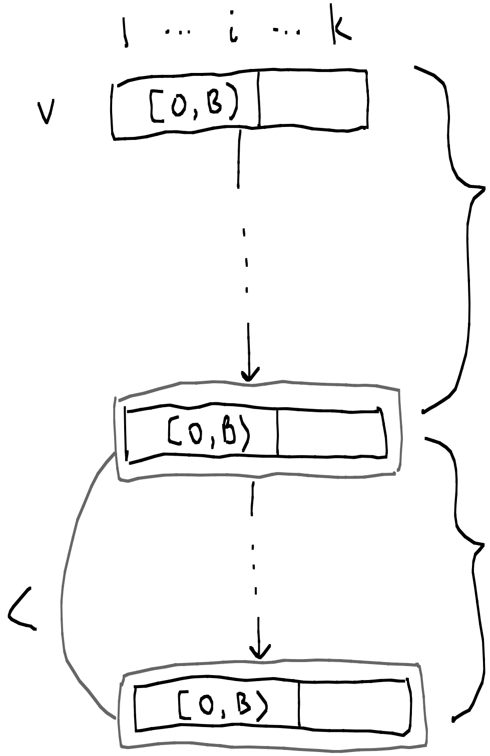
Boundedness

Key lemma for VAS [Rackoff]

If a computation as shown exists ($B \geq 2$),
then such a computation of length $\leq B^{|R|^c}$ exists.



Proof



length $\leq B^i$

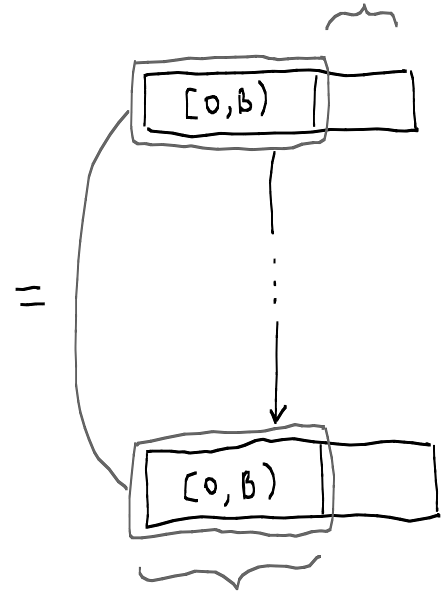
length $< (B^i+1)^2$

after removing

"simple loops"

effect on every place $i+1 \ldots k$

$\in [-B^i \max(R^-), B^i \max(R^+)]$



i-tuples

mutually distinct

Theorem [Borosh & Treybig AMS '76]

$$A \in (-m, m)^{k \times n}, \qquad b \in (-m, m)^{k}$$

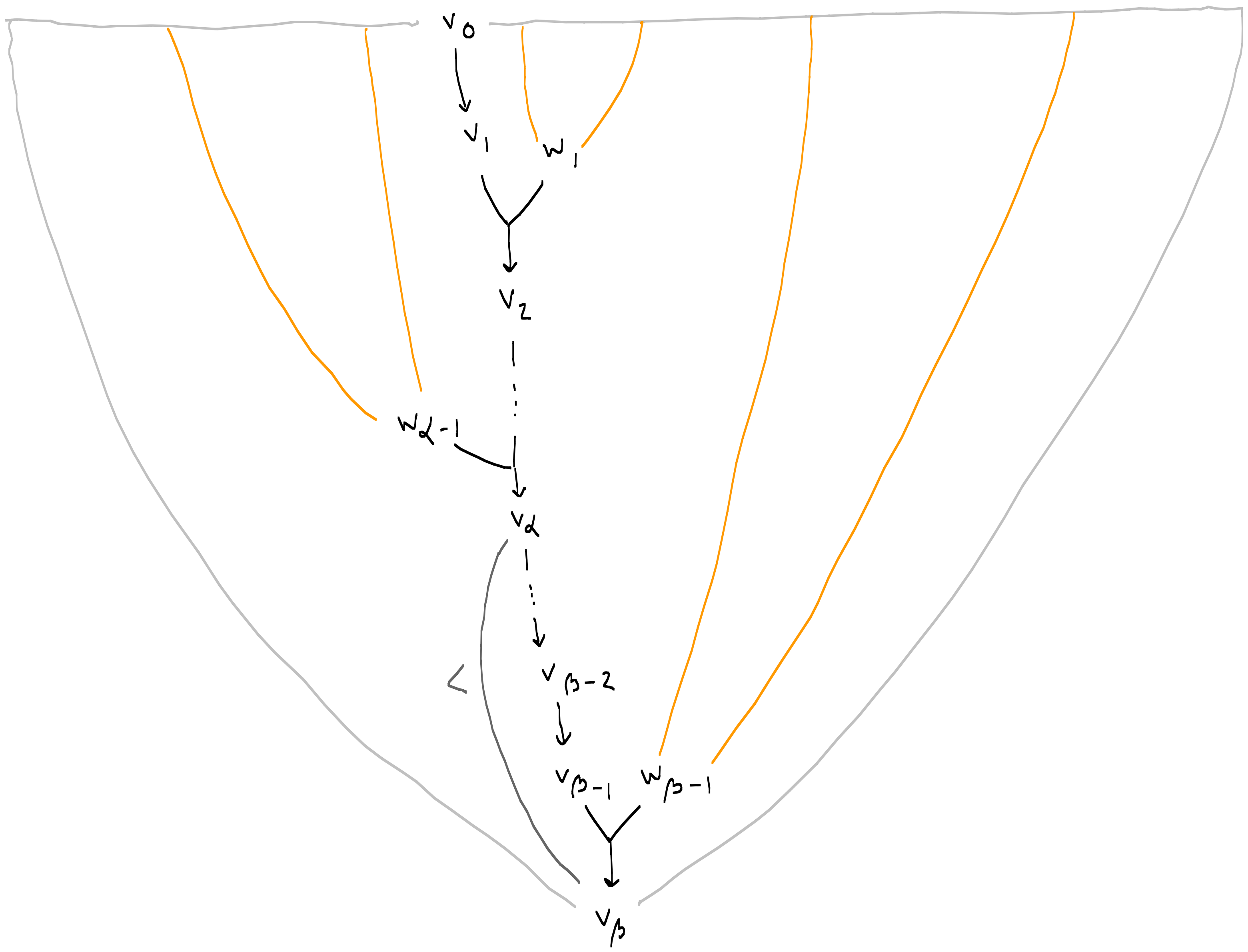
If $Ax \geq b$ for some $x \in \mathbb{N}^n$,

then $Ay \geq b$ for some $y \in \left[0, (\max\{n, m\})^{C' k}\right]^n$.

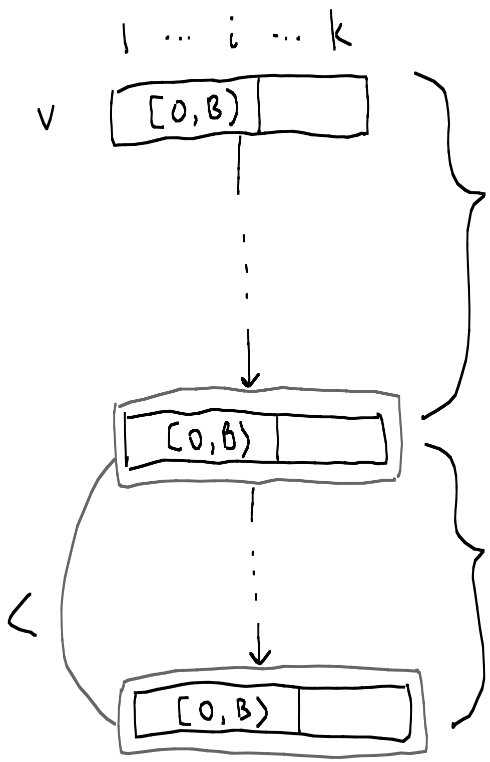
$$n \leq B^i \left(B^i \, 2^{|R|+1} + 1\right)^{k-i}$$

$$m \leq \left(B^i + 1\right)^2 2^{|R|}$$

Boundedness for BVAS



Proof



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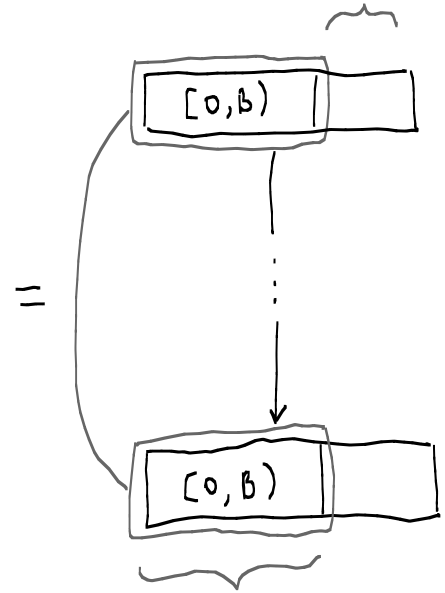
effect on every place $i+1 \dots k$

$\in \left[-B^i \max(R^-), \; B^i \underset{\infty}{\cancel{\max(R^+)}} \right]$



i-tuples

mutually distinct

Theorem ?

$$A \in (-m, \cancel{m}\ \infty)^{k \times n}, \qquad b \in (\cancel{-m}\ -\infty, m)^{k}$$

If $Ax \geq b$ for some $x \in \mathbb{N}^{n}$,

then $Ay \geq b$ for some $y \in [0, ?]^{n}$.

$$n \leq B^{i} \left(\cancel{B^{i} 2^{|R|+1} + 1)^{k-i}} \right.\ \infty$$

$$m \leq (B^{i} + 1)^{2}\ \cancel{2^{|R|}}\ \max(R^{-})$$